

CHAPTER-2

POINT GROUP

Aim:-

- (1) To calculate total no. of elements/operations
- (2) To generate point group.

(1)

C_{nh} Point group :-

$$C_{nh} = C_n + \sigma_h$$

Total no. of elements/operations = $2n$

$$C_{2h} :- 4 = C_{2h} = \frac{C_2}{1} + \frac{1\sigma_h}{1} + \frac{E}{1} + \frac{S_2}{1} \quad \leftarrow \text{no. of operations}$$

$$C_{3h} :- 6 = C_{3h} = \frac{C_3}{2} + \frac{1\sigma_h}{1} + \frac{E}{1} + \frac{S_6}{2}$$

$$C_{4h} :- 8 = C_{4h} = \frac{C_4}{3} + \frac{1\sigma_h}{1} + \frac{E}{1} + \frac{S_4 + S_2}{2}$$

$$C_{6h} :- 12 = C_{6h} = \frac{C_6}{4} + \frac{1\sigma_h}{1} + \frac{E}{1} + \frac{S_6}{4}$$

Note 1 In case of C_{nh} point group, we will try improper axis (S_n):

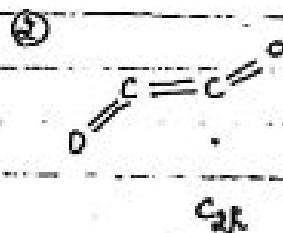
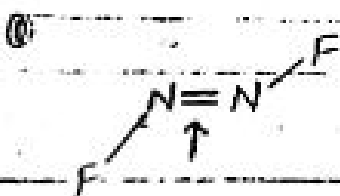
If $n = \text{even}$ $\xrightarrow{\text{no. of operation}}$ S_n and/or $\frac{S_n}{2}$

If $n = \text{odd}$ $\xrightarrow{\quad}$ S_n

Note 2 No. of operations in improper axis

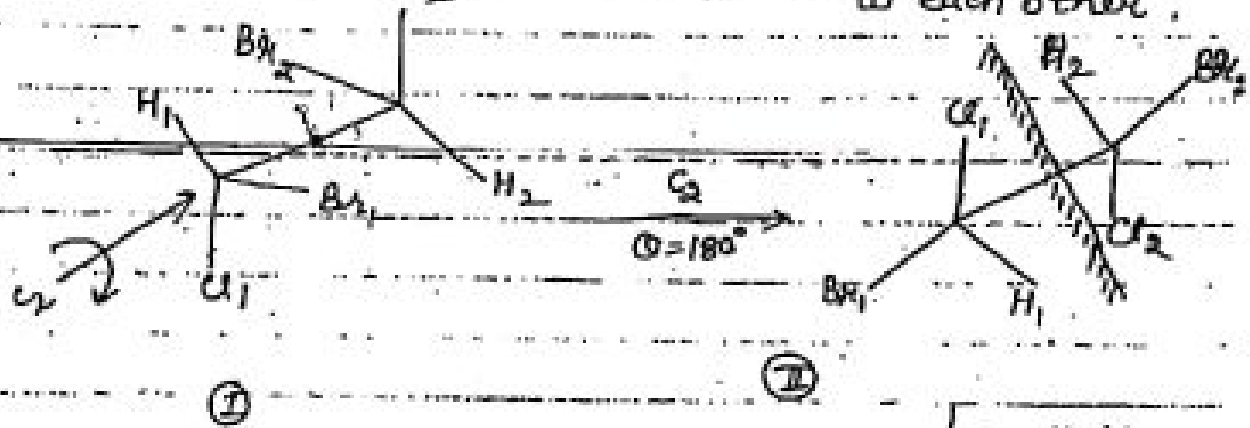
If $n = \text{even}$, then no. of operations = $\frac{n}{2}$

If $n = \text{odd}$, " " " " = $n-1$

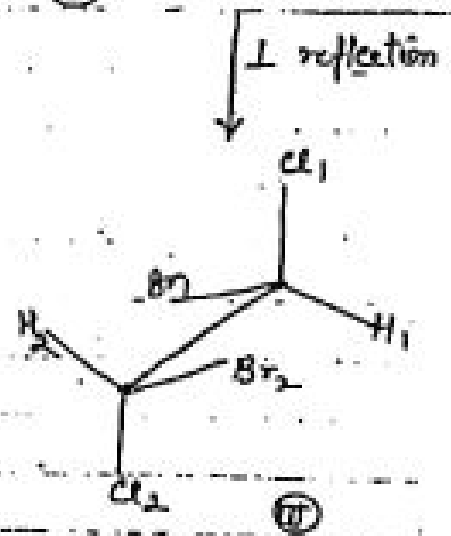


$S_2 = C_2 + \perp \text{ plane}$
 $\bullet = C_2 + \perp \text{ plane}$

produce equivalent orientation molecules having this type of symmetry requires atoms lying outside this pt. & diagonally placed to each other.



Ⓘ & ⓓ are equivalent orientation.

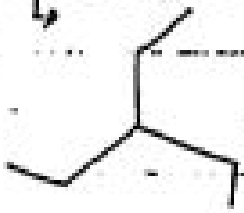


* IDENTITY (E) :

Nothing

Grate-2011

3



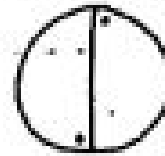
C_{2h}

4



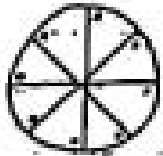
C_{4h}

5



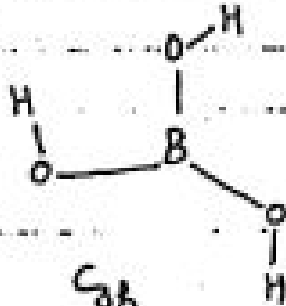
C_{2h}

6



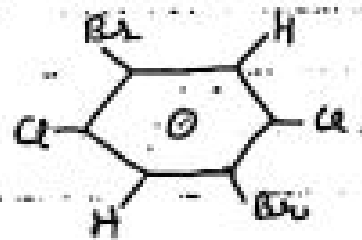
C_{6h}

7



C_{3h}

8



C_{2h}

9



C_{2h}

10



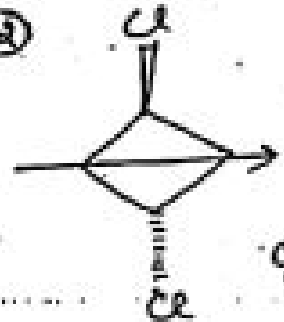
C_{2h}

11



C_{2h}

12



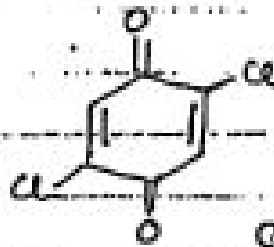
C_{2h}

13



C_{2h}

14



C_{3h}

Q) Cnv Point group:

$$C_{nv} = C_n + n\sigma_v$$

No. of operations = $2n$

~~$C_{2v}: 8 = C_2 + 2\sigma_v: 1+2$~~

~~$C_{3v}: 6 = C_3 + 3\sigma_v$~~

~~$C_{4v}: 8 = C_4 + 4\sigma_v$~~

~~$C_{5v}: 10 = C_5 + 5\sigma_v$~~

$$4, 2 \quad C_{2v} = \frac{C_2 + 2\sigma_v + E}{1 \quad 2 \quad 1}$$

$$6 = C_{3v} = \frac{C_3 + 3\sigma_v + E}{2 \quad 3 \quad 1}$$

$$8 = C_{4v} = \frac{C_4 + 4\sigma_v + E}{3 \quad 4 \quad 1}$$

$$10 = C_{5v} = \frac{C_5 + 5\sigma_v + E}{4 \quad 5 \quad 1}$$



C_{2v}



$C_{\infty v}$



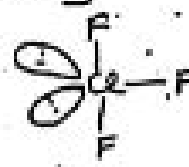
C_{2v}



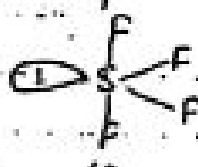
(See also) C_{2v}



C_{3v}



C_{2v}



(See saw) C_{2v}



C_{4v}



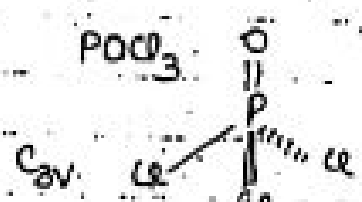
$C_{\infty v}$



C_{2v}



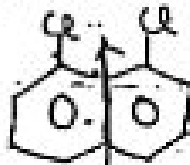
C_{2v}



C_{3v}



C_{2v}



C_{2v}



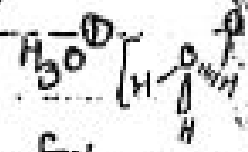
C_{2v}



C_{3v}

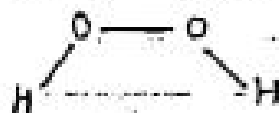


C_{3v}



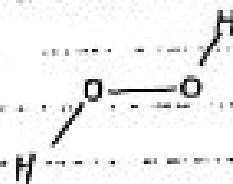
C_{3v}

* Point group of H_2O_2 :-



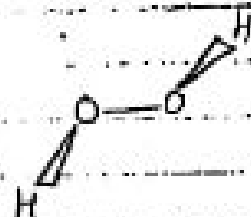
cis- H_2O_2

C_{2v}



trans- H_2O_2

C_{2h}



open-book

C_2

Q:- Point group of H_2O_2 is

- i) C_{2v} ii) C_{2h} iii) C_2 iv) D_{2h}

Q:- Which one of the following is not point group of $H_2O_2 \rightarrow D_{2h}$

Q:- Point group of trans- $H_2O_2 \rightarrow C_{2h}$

Q:- " " " cis- " $\rightarrow C_{2v}$

(3) D_{2h} Point group :-

Formula: $C_n + nC_2 + n\sigma_v + I - R$

No. of operations = $4n$

$$8 = D_{2h} = \frac{C_2}{1} + \frac{2C_2}{2} + \frac{2\sigma_v}{2} + \frac{1\sigma_h}{1} + E + \frac{S_2}{1}$$

$$12 = D_{3h} = \frac{C_3}{2} + \frac{3C_2}{3} + \frac{3\sigma_v}{3} + \frac{1\sigma_h}{1} + E + \frac{S_6}{2}$$

$$16 = D_{4h} = \frac{C_4}{3} + \frac{4C_2}{4} + \frac{4\sigma_v}{4} + \frac{1\sigma_h}{1} + E + \frac{S_4}{2} + \frac{S_2}{1}$$

$$20 = D_{5h} = \frac{C_5}{4} + \frac{5C_2}{5} + \frac{5\sigma_v}{5} + \frac{1\sigma_h}{1} + E + \frac{S_5}{4}$$

Note - (3) In case of D_{nh} pt. group, we will try S_n operation

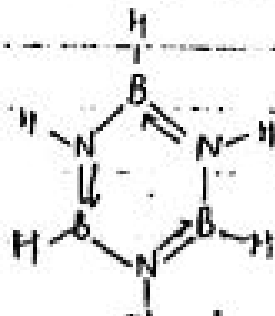
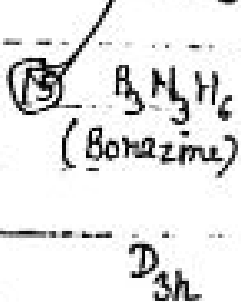
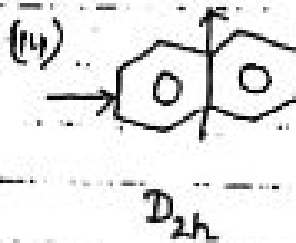
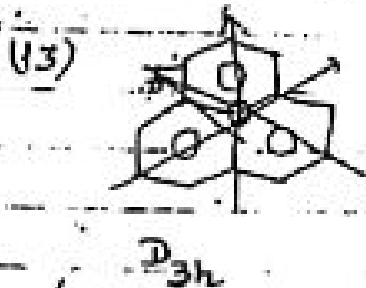
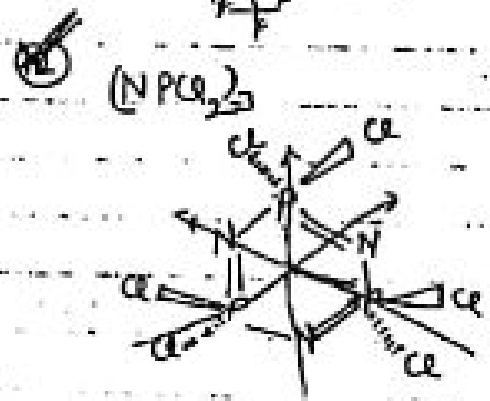
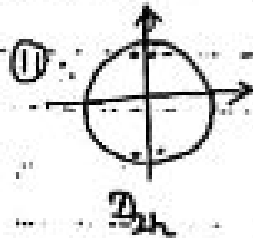
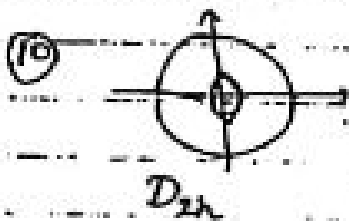
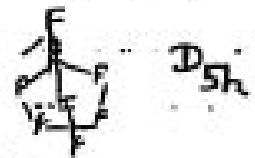
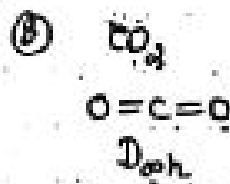
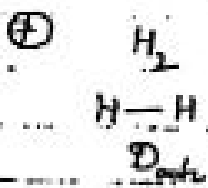
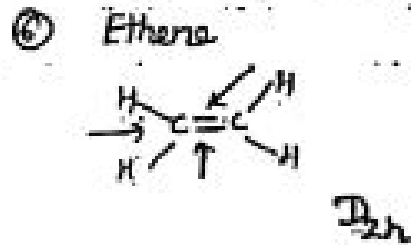
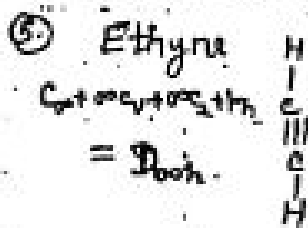
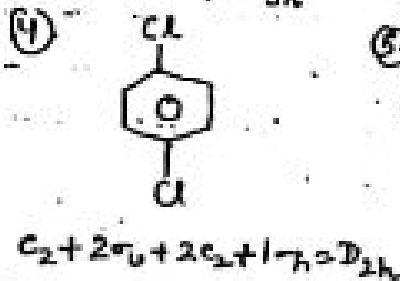
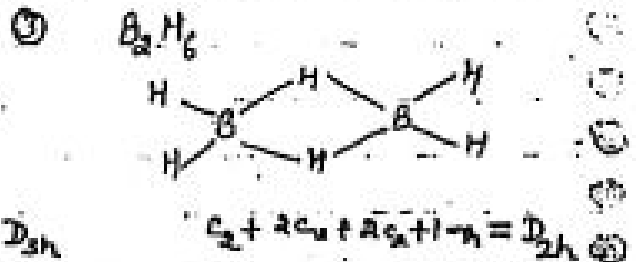
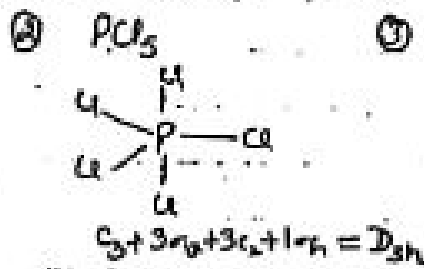
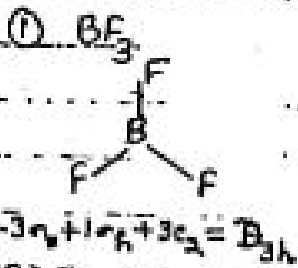
if $n = \text{even}$ then S_n and $S_{n/2}$ operations

$n = \text{odd}$ " S_n operations = S_{n-1}

Note-4 No. of operations in improper axis

$\frac{n}{2}$ if $n = \text{even}$ $\frac{n}{2}$ formula

$n - 1$ if $n = \text{odd}$ $n - 1$ formula



⇒ Finding out no. of operations in improper axis :-

Prove.

(a) when $n = \text{even}$, no. of operation = $\frac{n}{2}$

i) S_2 : $n = 2$ $\therefore \frac{2}{2} = 1$

8. $C_2^1 \cdot \sigma^1 = S_1^1$
 $C_2^2 \cdot \sigma^2 = E \cdot E = E$ } no. of operation = 1.

(ii) S_4



$C_4^1 \cdot \sigma^1 = S_4^1$
 $C_4^2 \cdot \sigma^2 = C_2^1 \cdot E = C_2^1$
 $C_4^3 \cdot \sigma^3 = C_4^3 \cdot \sigma = S_4^3$
 $C_4^4 \cdot \sigma^4 = E \cdot E = E$ } No. of operation = 2

(iii) S_6

$C_6^1 \cdot \sigma^1 = S_6^1$
 $C_6^2 \cdot \sigma^2 = C_3^1 \cdot E = C_3^1$
 $C_6^3 \cdot \sigma^3 = C_2^1 \cdot \sigma = S_2^1(i)$
 $C_6^4 \cdot \sigma^4 = C_3^2 \cdot E = C_3^2$
 $C_6^5 \cdot \sigma^5 = C_6^5 \cdot \sigma = S_6^5$
 $C_6^6 \cdot \sigma^6 = E \cdot E = E$ } no. of operations = 3

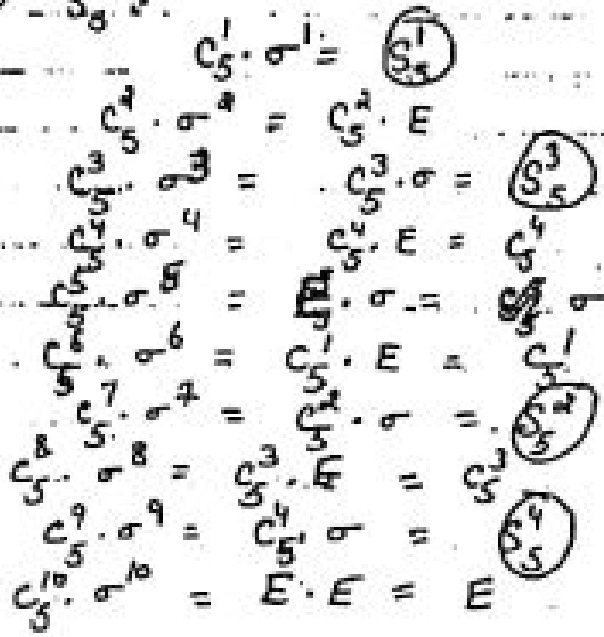
Note: If n is even, repetition of sequence occurs from S_n^{th}

(b) if $n = \text{odd}$, no. of operations = $n-1$

i) S_3

$C_3^1 \cdot \sigma^1 = S_3^1$
 $C_3^2 \cdot \sigma^2 = C_3^2 \cdot E = C_3^2$
 $C_3^3 \cdot \sigma^3 = E \cdot \sigma = \sigma$
 $C_3^4 \cdot \sigma^4 = C_3^1 \cdot E = C_3^1$
 $C_3^5 \cdot \sigma^5 = C_3^2 \cdot \sigma = S_3^2$
 $C_3^6 \cdot \sigma^6 = E \cdot E = E$ } No. of operation = 2

(ii) S_6

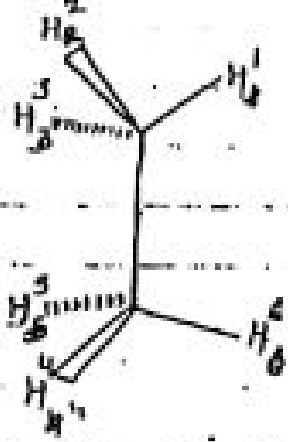


No. of operations = 12

Note:-

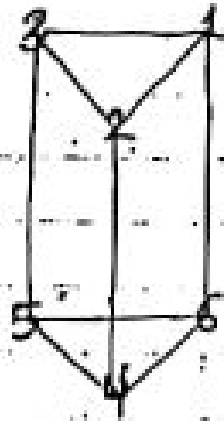
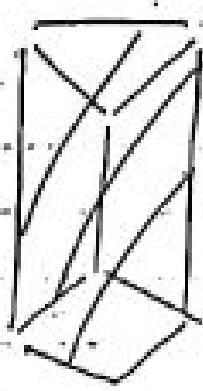
if n odd, Repeation of Sequence occurs from S_{2n+1}

→ Point group of eclipsed ethane : D_{3h}



→ H_1 is eclipsed H_4
 H_2 " " H_5
 H_3 " " H_6

→ S_6 axis passing through C-C bond and interchanging $H_1 \rightarrow H_2 \rightarrow H_3$ & $H_4 \rightarrow H_5 \rightarrow H_6$



→ 1st C_2 axis passing through middle of C-C bond (H_1-H_4 edge) and interchanging H_1 with H_4 , H_2 with H_5 and H_3 with H_6

→ 2nd C_2 axis passing through e. middle of C-C bond (H_2-H_5 edge)

and interchanging H_2 with H_5 , H_1 with H_4 & H_3 with H_6

⇒ 3rd C_2 -axis passing through middle of C-C bond (H^3-H^5 edge) and interchanging H^3 with H^5 , H^1 with H^4 & H^2 with H^6 .

⇒ 1st vertical plane (σ_v) bisecting $H^1-C-C-H^6$ & reflecting H^2 with H^3 & H^4 with H^5 .

⇒ 2nd σ_v bisecting $H^2-C-C-H^4$ & reflecting H^1 with H^3 & H^5 with H^6 .

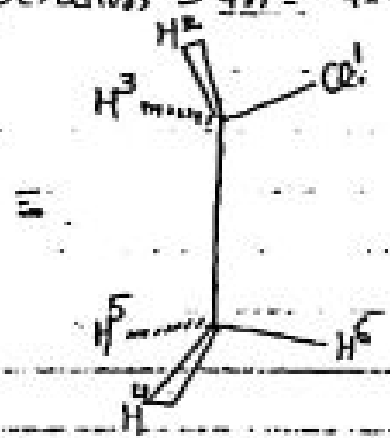
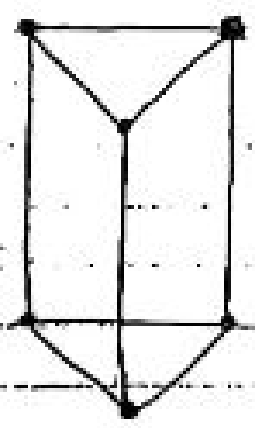
⇒ 3rd σ_v bisecting $H^3-C-C-H^5$ & reflecting H^1 with H^2 & H^4 with H^6 .

⇒ 1 σ_h bisecting C-C bond & reflecting H^1 with H^6 , H^2 with H^4 & H^3 with H^5 .

$$C_3 + 3C_2 + 3\sigma_v + 1\sigma_h = D_{3h}$$

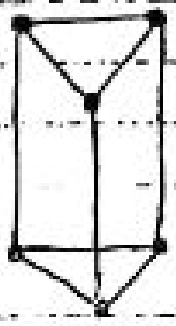
$$\text{No. of operation} = 4n = 4 \times 3 = 12$$

(a) H^1 is replaced by Cl^1 .

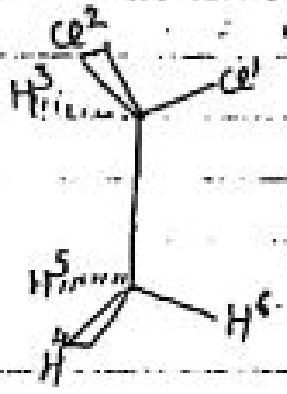


$$\sigma \Rightarrow C_3$$

(b) H^1 is replaced by Cl^1 & H^2 with Cl^2 :

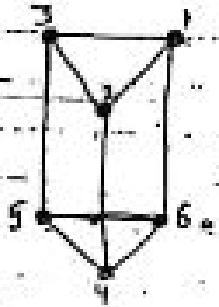


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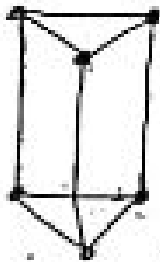
$$\sigma \Rightarrow C_3$$

(c) H^3 is also replaced by Cl^3 :

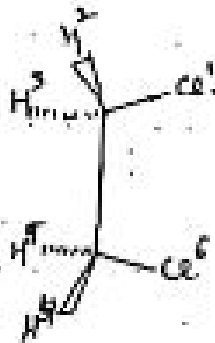


$$C_3 + 3\sigma_v = \boxed{C_{3v}}$$

(d) H^1 & H^6 are replaced by Cl^1 's:



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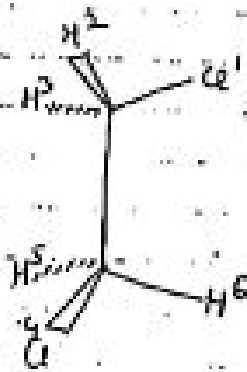


$$C_2 + 2\sigma_v = \boxed{C_{2v}}$$

(e)

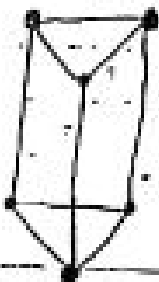


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$$\boxed{C_2}$$

(f)



=



$$\boxed{C_3}$$

Eclipsed Ethane

If one substitution = C_2

two " " of same carbon = C_2

three " " " " = C_{3v}

two " " different C of same edge = C_{2v}

two " " " " " " different " " = C_2

(4) D_{2d} Point group

$$C_n + nC_2 + n\sigma_d$$

$$\text{No. of operations} = 4n$$

$$8 = D_{2d} = \frac{C_8}{1} + 2\frac{C_2}{2} + 2\frac{\sigma_d}{2} + E + \frac{S_8}{2}$$

$$12 = D_{3d} = \frac{C_3}{2} + 3\frac{C_2}{3} + 3\frac{\sigma_d}{3} + E + \frac{S_6}{2} + \frac{S_6}{2}$$

(excluding)

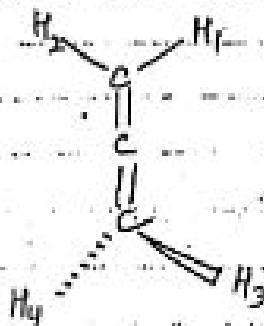
$$16 = D_{4d} = \frac{C_4}{2} + 4\frac{C_2}{4} + 4\frac{\sigma_d}{4} + E + \frac{S_8}{4}$$

Note:- In case of D_{2d} point group, we will try same operations

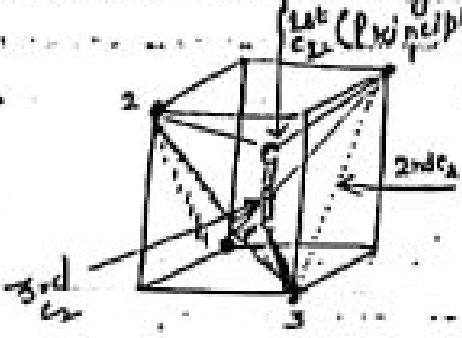
Examples:

Allene

⇒ Point group of Allene = $D_{2d} = C_2 + 2C_2 + 2\sigma_d$



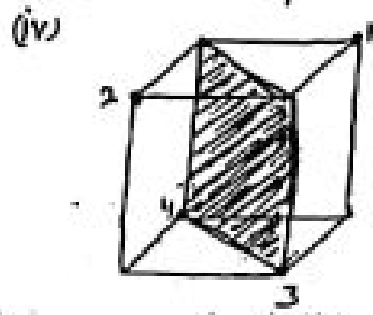
Opposite faces are squares.
Other side faces are rectangles.
Let C_2 (Principal axis) be passing through max no. of



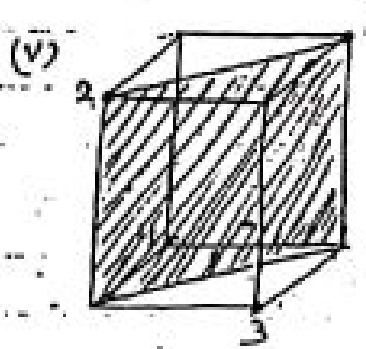
(i) 1st C_2 axis passing through (1-2) diagonal and outgoing through (3-4) diagonal & interchanging H_1 with H_2 & H_3 with H_4 .

(ii) 2nd C_2 -axis entering through (1-3) diagonal and outgoing through (2-4) diagonal & interchanging H_1 with H_3 & H_2 with H_4 .

(iii) 3rd C_2 -axis entering through (2-3) diagonal and outgoing through (1-4) diagonal & interchanging H_2 with H_3 & H_1 with H_4 .



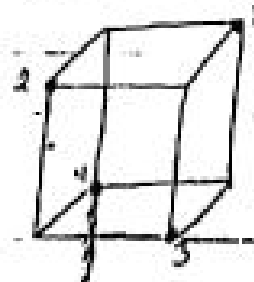
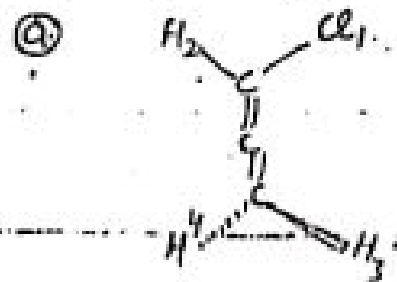
1st dihedral plane (σ_d) bisecting H_3 and H_4 and 3 carbon atoms and reflecting H_1 with H_2 .



2nd σ_d bisecting H_1 & H_2 and 3 carbon atoms and reflecting H_3 with H_4 .

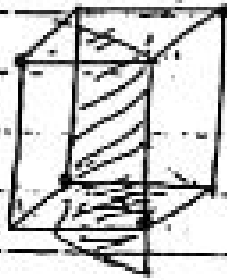
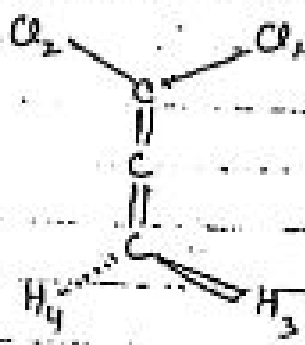
$$C_2 + 2C_2 + 2\sigma_d \rightarrow D_{2d}$$

only one plane (σ)



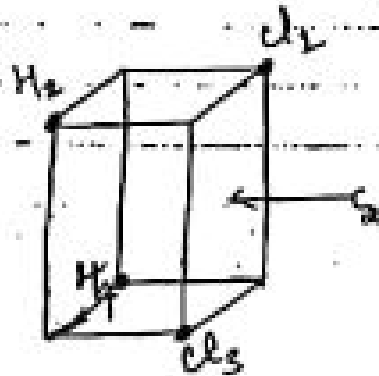
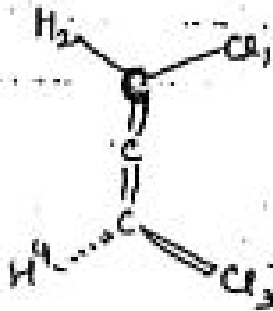
C_s

(b)



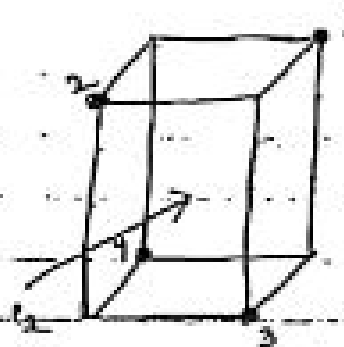
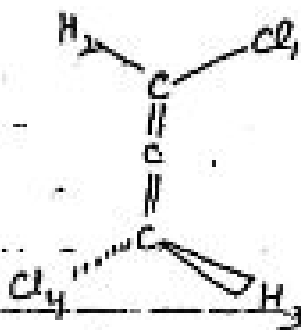
$$C_2 + \sigma_v = C_{2v}$$

(c)



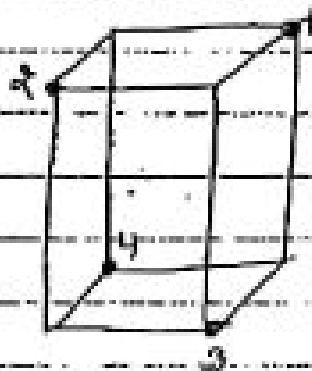
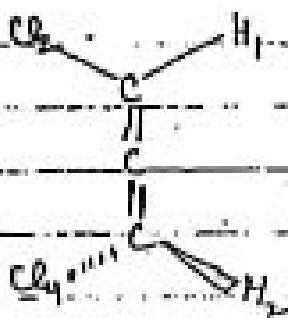
$$C_2$$

(d)



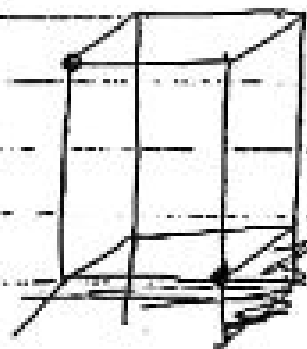
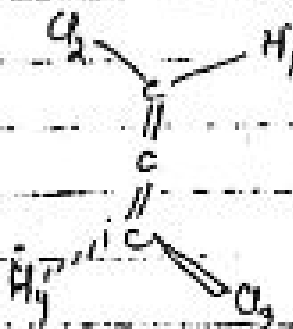
$$C_{2v}$$

(e)



$$C_{2v}$$

(f)



$$C_{2v}$$

* Conclusion \Rightarrow * If one H of allene is substituted by Cl Point gr: is C_2 .

* If 2 H's of same carbon is substituted $\rightarrow C_{2v}$

* " " " " different " " " " $\rightarrow C_2$

Qo :- For this molecule $CH_2=C_x=CH_2$, $x=1, 2, 3, \dots, n$

② No. of operations does not depend upon value of x . True

③ Proof :- If $x = \text{odd}$ (behave like allene)

$$D_{2d} = 4n = 8$$

If $x = \text{even}$ (behave like ethene)

$$D_{2h} = 4n = 8$$

④ Improper axis present is same for all values of x . False

If $x = \text{odd}$, $D_{2d} = S_4$

" $x = \text{even}$, $D_{2h} = S_4$

⑤ There is no change in no. of operations if ² H's of same carbon is replaced by F for all values of x . True

If $x = \text{odd}$ (Allene)



C_{2v} & No. of operations = 4

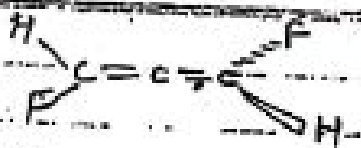
If $x = \text{even}$ (alkene)



C_{2v} & No. of operations = 4

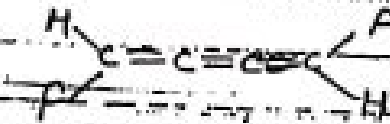
⑥ There is no change in no. of operations if two H's of different carbon are substituted by F for all values of x . False

If $x = \text{odd}$



$\Rightarrow C_2$ & No. of operations = 2

If $x = \text{even}$



$\Rightarrow C_{2v}$ & No. of operations = 4

\Rightarrow Staggered Ethane : $D_{3d} \Rightarrow C_3 + 3C_2 + 3\sigma_d$

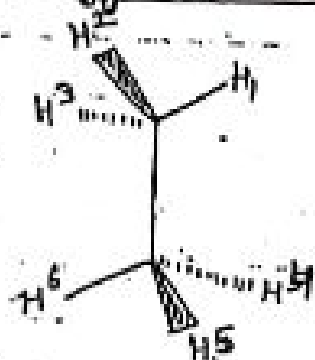


Figure-1

H^1 is anti-staggered to H^6
 H^2 " " " " " "
 H^3 " " " " " "

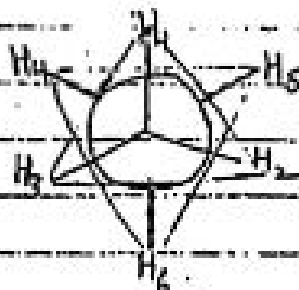


Figure-2

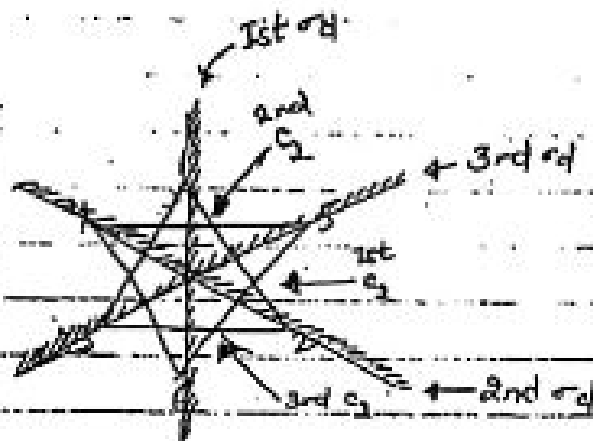
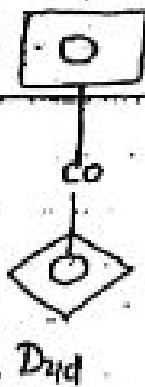
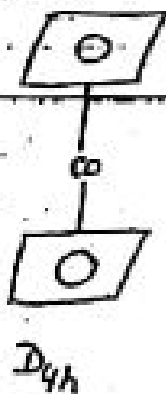


Figure-3

Or C_3 axis passing through C-C bond and interchanging $H^1 \rightarrow H^2 \rightarrow H^3$ & $H^4 \rightarrow H^5 \rightarrow H^6$.

- (i) 1st C_2 -axis interchanging H^1 with H^6 , H^2 with H^5 & H^3 with H^4
- (ii) 2nd " " " " " " H^1 with H^5 , H^2 with H^4 & H^3 with H^6
- (iii) 3rd " " " " " " H^1 with H^4 , H^2 with H^6 & H^3 with H^5
- (iv) 1st σ_d bisecting $H^1-C-C-H^4$ & reflecting H^2 with H^3 & H^5 with H^6
- (v) 2nd " " " " " " $H^1-C-C-H^5$ & " " " " H^2 with H^6 & H^3 with H^4
- (vi) 3rd " " " " " " $H^1-C-C-H^6$ & " " " " H^2 with H^4 & H^3 with H^5

Point group of substituted staggered Ethane +

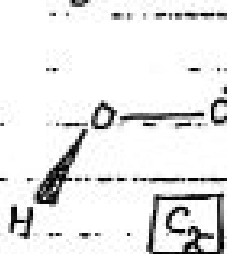


⑤ C_n Point group = $E_n + E$
 No. of operations = n

$C_2 = C_2 + E = 2$

$C_3 = C_3 + E = 3$

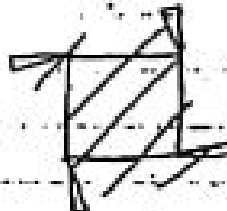
(i) H_2O_2



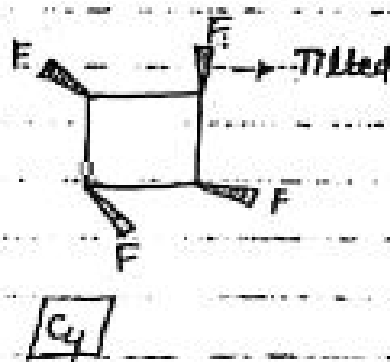
(ii) PPh_3



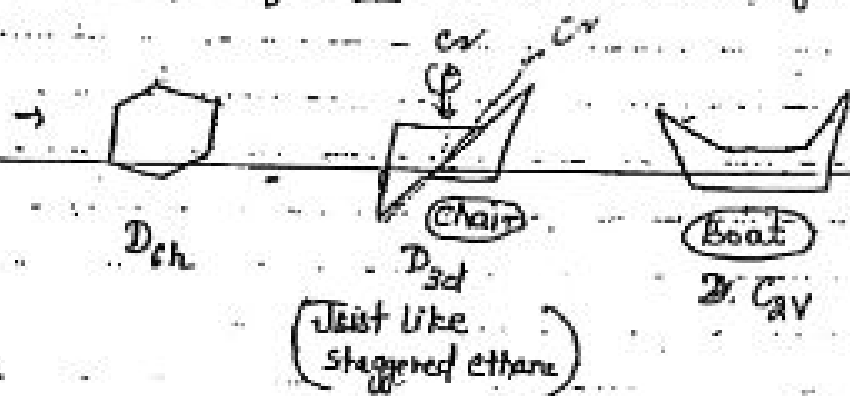
(iii)



(iv)



→ Example of D_2 :- Twisted boat forms of cyclohexane.



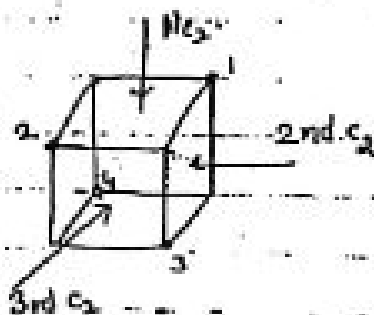
★ Tetrahedral Point group :- CH_4

- ① 4 corners ② 6 Edges ③ 4 triangular faces

Total no. of operations = 24. $(8 + 3 + 6 + 6 + 1 = 24)$

① 4 C_2 axis passing through each C-H bond through each corner. \Rightarrow 8 operations.

② 3 C_2 axis passing through opposite edge or faces of cube. \Rightarrow 3 operations

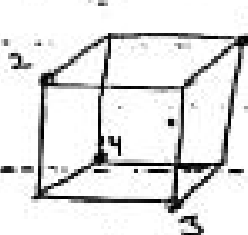


③ 3 S_4 along each C_2 axis \Rightarrow 6 operations

④ 6 σ_d (dihedral plane) \Rightarrow 6 operation

⑤ Identity (E) \Rightarrow 1 operation.

⇒ How there are 6 dihedral planes :-



Bisecting	Reflecting
1, 2	3 bisect 4
1, 3	2 bis 4
1, 4	2 bis 3
2, 3	1 bis 4
2, 4	1 bis 3
3, 4	1 bis 2

$$9 + 8 + 6 + 6 + 8 + 3 + 6 + 1 + 1 = 48$$

⇒ Octahedral Point group :- SF_6 [No. of operations = 48]

6 corners, 12 edges, 8 triangular faces ⇒ 8

1. 3- C_4 axis passing through opposite corners ⇒ 6
2. 4- C_3 axis passing through opposite triangular faces ⇒ 8
3. 6- C_2 axis passing through opposite edges ⇒ 6
4. 3- S_4 axis along each C_4 axis ⇒ 6
5. 4- S_6 axis along each C_3 axis ⇒ 12 ⇒ 8 (Excluding i)
6. 3- σ_h perpendicular to each C_4 axis ⇒ 3
7. 6- σ_d (dihedral planes) ⇒ 6
8. 1- i (Inversion centre) ⇒ 1
9. 1-E (Identity) ⇒ 1